A Stable Iterative Approach for 3D Electromagnetic Full Wave Analysis Based on Wave Equation

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Abstract — This paper reports the convergence property of a stable iterative approach to solving a 3D time-harmonic full wave electromagnetic problem based on the vector wave propagation equations. This approach utilizes a time domain update scheme of the vector wave equation FDTD analysis for solving a steady state fixed point in the time-harmonic domain. Numerical experiments show that the formulation has an asymptotical convergence property for large-scale EM problems.

I. Introduction

A numerical electromagnetic analysis such as edge-based FEM is an efficient tool to investigate arbitrarily shaped 3D complex structures. Large null space of the coefficient matrix, however, causes difficulties in convergence when iterative algorithms are applied [1]. Various approaches have been studied to improve this slow convergence issue by preconditioning the original finite element matrix [1][2][3]. The large-scale electromagnetic problem, however, still suffers from the convergence difficulties due to the 'low frequency errors' [4] and needs more effective algorithms. Recently a new iteration scheme for the FDFD formulation was proposed and proved to converge to a fixed point under some conditions [5][6]. This approach utilizes a time domain update scheme of the FDTD algorithm. On following this approach, we investigated a stable iterative algorithm for a time-harmonic 3D full wave analysis which is based on the vector wave equation FDTD [7].

II. Formulation

A time-harmonic boundary value problem for the vector wave equation that governs electromagnetic phenomena is expressed by the electric field intensity vector $\boldsymbol{E}$ as follows:

$$\nabla\times\frac{1}{\mu_r}\nabla\times\boldsymbol{E}+j\omega\mu_0\sigma\boldsymbol{E}-\omega^2\varepsilon_0\varepsilon_r=-j\omega\boldsymbol{J}\quad\text{in }\Omega \qquad (1)$$

$$\boldsymbol{E}\times\boldsymbol{n}=0\ \text{on}\ \Gamma_d \qquad (2)$$

where Ω is a finite three dimensional domain with a Diriclet-type boundary Γ_d and outward normal vector $\boldsymbol{n}$ [8]. $\mu_0, \mu_r, \varepsilon_0, \varepsilon_r, \sigma, \omega$ and $\boldsymbol{J}$ are the magnetic permeability of free space, relative magnetic permeability, the electric permittivity of free space, relative electric permittivity, conductivity, angle frequency and the impressed volume electric current density, respectively. Using a central difference approximation for space with Cartesian grids, the following finite different system is obtained:

$$M\ \boldsymbol{x}=-j\omega\boldsymbol{f} \qquad (3)$$

where M is the finite difference matrix, $\boldsymbol{x}$ denotes the vector of the unknown expansion coefficients of $\boldsymbol{E}$ and $\boldsymbol{f}$ is the discretized term associated with the imposed source current $\boldsymbol{J}$ [8]. The electric field E_z is, for example, expressed by the surrounding field variables as follows:

$$\begin{aligned}&\frac{E_x^{ijk+1}-E_x^{ijk}-E_x^{i-1jk+1}+E_x^{i-1jk}}{\mu_{r_y}\Delta z\Delta x}\\&+\frac{2E_z^{ijk}-E_z^{i+1jk}-E_z^{i-1jk}}{\mu_{r_y}\Delta x^2}\\&+\frac{2E_z^{ijk}-E_z^{ij+1k}-E_z^{ij-1k}}{\mu_{r_x}\Delta y^2}\\&+\frac{E_y^{ijk+1}-E_y^{ijk}-E_y^{ij-1k+1}+E_y^{ij-1k}}{\mu_{r_y}\Delta y\Delta z}\\&+j\omega\mu_0\sigma E_z-\omega^2\mu_0\varepsilon_0\varepsilon_z E_z=-j\omega\mu_0 J_z.\end{aligned} \qquad (4)$$

This linear system is ill-conditioned when the degree of freedom becomes large. Therefore special measures are required for quick convergence by iterative solvers.

III. Iteration Scheme

Converting the expression (4) into the time domain, the following update scheme is obtained [7]:

$$\begin{aligned}E_z^{ijk(n+1)}&=\frac{4\varepsilon_z}{2\varepsilon_z+\sigma\Delta t}E_z^{ijk(n)}-\frac{2\varepsilon_z-\sigma\Delta t}{2\varepsilon_z+\sigma\Delta t}E_z^{ijk(n-1)}\\&-\frac{\Delta t}{2\varepsilon_z+\sigma\Delta t}(J_z^{n+1}-J_z^{n-1})-\frac{2\Delta t^2}{2\varepsilon_z+\sigma\Delta t}\\&\times(\frac{E_x^{ijk+1}-E_x^{ijk}-E_x^{i-1jk+1}+E_x^{i-1jk}}{\mu_{r_y}\Delta z\Delta x}\\&+\frac{2E_z^{ijk}-E_z^{i+1jk}-E_z^{i-1jk}}{\mu_{r_y}\Delta x^2}\\&+\frac{2E_z^{ijk}-E_z^{ij+1k}-E_z^{ij-1k}}{\mu_{r_x}\Delta y^2}\\&+\frac{E_y^{ijk+1}-E_y^{ijk}-E_y^{ij-1k+1}+E_y^{ij-1k}}{\mu_{r_y}\Delta y\Delta z}).\end{aligned} \qquad (5)$$

Suppose that the iteration process achieves a steady-state in the frequency domain, E^{n+1}_z, E^n_z and E^{n-1}_z can be expressed as $e^{j\omega\Delta t}E_z^{(f)}, E_z^{(f)}$ and $e^{-j\omega\Delta t}E_z^{(f)}$, respectively. It is easily shown that E_z^f satisfy (4) when $\Delta t\to 0$ [5].

IV. Numerical Example

Numerical examples are performed to investigate the convergence property of this iterative wave equation-based formulation. The following first order absorbing boundary condition [8] is imposed on the outer boundary:

$$\frac{\partial E_i}{\partial i} = -jkE_i, \; i=x, y, z. \tag{6}$$

TABLE I
SIMULATION STATISTICS

Number of elements	3,375,000
Number of freedom	9,990,450
Absorbing boundary condition	1st order absorbing boundary condition
Element type	Cartesian grid
Frequency	500(MHz)
Relative Permittivity	4.7 (Dielectric material)
Conductivity(S/m)	5.76e+7 (Metal)

A. Model

A parallel plate resonator shown in Fig. 1 is analyzed with a voltage excitation at the origin in Z-direction. Simulation statistics of the benchmark analysis is listed in Table I. Field variables $E_z^{i,j,k(n)}$ are observed on the same plane of the input stimulus at (X,Y,Z)=(10,10,0) and (100,100,0).

B. Convergence Property

Fig. 2 shows the convergence profiles at the observation points. After some iterations we can observe the asymptotic convergence of the field intensities to each fixed point.

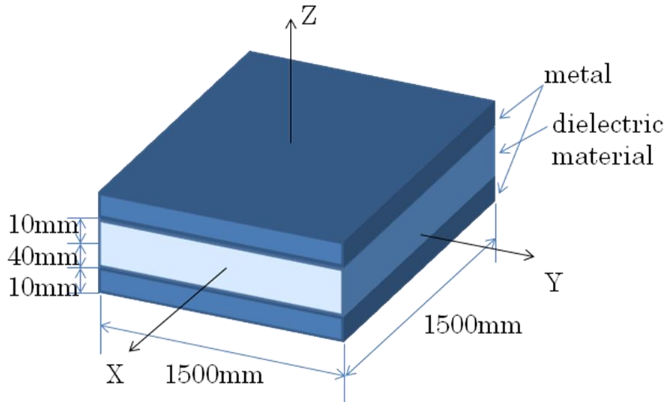

Fig. 1. Benchmark model

This property is preserved when the input frequency is changed to 1GHz. The same results are obtained in some other benchmark model cases.

C. Discussion

In full wave numerical electromagnetics, a large-scale problem has intrinsic convergence difficulties [3][4], especially with more than 1,000,000 unknowns. This approach based on the time-domain update scheme, however, showed an asymptotic but stable convergence profile with the problem of nearly 10,000,000 unknowns. Since this scheme is based on an ever heavily studied time domain update scheme, the robustness and stability can be expected for the problems consists of arbitrarily shaped complex electromagnetic structures. By taking average of the data sequence, the steady-state field values can be estimated and utilized as an initial guess for another iteration algorithm. We now try to apply this scheme to more complex electromagnetic structures.

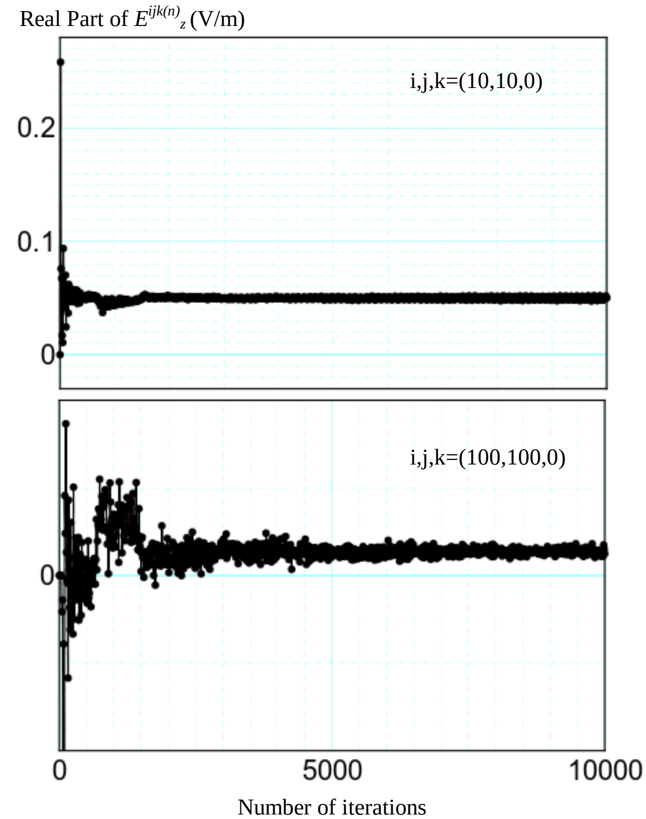

Fig. 2. Convergence profiles (Top: at (10,10,0), Bottom: at(100,100,0))

V. References

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